θ	0	40	80	120	180
σ_θ	+10.87	+8.06	+6.97	−4.55	−1.58
σ_θ	+ 5.52	+3.96	+2.84	−3.12	−4.28

Values of σ_θ for the isotropic case are given in the third line; θ is the angle measured from the normal to the half-plane boundary.

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AXISYMMETRIC IMPRESSION OF TWO STAMPS INTO AN ELASTIC SPHERE

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The problem of the impression of two identical axisymmetric stamps in an elastic sphere is considered. It is assumed that the surface of the sphere outside the stamps is stress-free, while there are no shear stresses under the stamps. A solution is obtained for arbitrary stamps for both given and unknown in advance boundaries of the contact domains by the method elucidated in [1]. A numerical calculation is presented for spherical stamps under internal contact with the sphere.

The contact problem for a sphere in such a formulation (when the boundaries of the contact domains are known) was first studied in [2]. The problem was reduced to determining certain coefficients from dual series-equations containing Legendre polynomials. The method permitting reduction of the solution of the obtained dual series-equations to the solution of an infinite system of linear algebraic equations is indicated. This method is reduced to an integral equation of the first kind in [3] and a possible scheme is indicated for the approximate solution of the equation obtained.

1. Let us consider the contact problem of impressing two axisymmetric stamps (Fig. 1), whose surface is given in a spherical r, θ, φ coordinate system by the equation

$$r = R\,[1 + \rho(\theta)],\quad \rho(\pi - \theta) = \rho(\theta),\quad \rho(0) = 0 \tag{1.1}$$

onto an elastic sphere $r \leqslant R$.

The boundary conditions (on the sphere $r = R$) are ($2aR$ is the approach of the stamps):

$$u_r = R\,[-a\,|\cos\theta| + \rho(\theta)],\quad 0 \leqslant \theta \leqslant \gamma \ \text{ and } \ \pi - \gamma \leqslant \theta \leqslant \pi \tag{1.2}$$

$$\sigma_r = 0, \qquad \gamma < \theta < \pi - \gamma$$
$$\tau_{r\theta} = 0, \qquad 0 \leqslant \theta \leqslant \pi$$

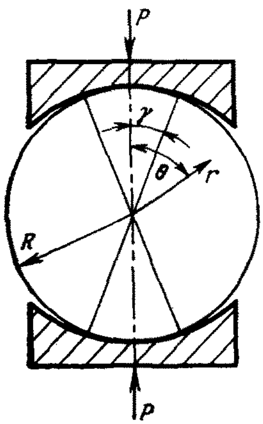

Fig. 1

Determine the contact pressure $\sigma(\theta)$ $(0 \leqslant \theta \leqslant \gamma)$ and (in the case of stamps without corners) the magnitude of the angle γ characterizing the size of the contact domains.

Using the closed solution of the problem of equilibrium of an elastic sphere [4], we satisfy conditions (1.2). Taking this into account, we obtain the integral equation

$$\int_0^\gamma \sigma(\alpha)\left[H(\theta, \alpha) + H(\theta, \pi - \alpha)\right] \sin \alpha \, d\alpha = 2\pi G \times \tag{1.3}$$
$$\left[-a\,|\cos\theta| + \rho(\theta)\right], \quad 0 \leqslant \theta \leqslant \gamma$$

to determine the contact pressure by virtue of the symmetry $\sigma(\pi - \theta) = \sigma(\theta)$, or after a change of variables

$$\varepsilon t = \operatorname{tg} {}^1/_2\,\alpha, \quad \varepsilon x = \operatorname{tg} {}^1/_2\,\theta, \quad \varepsilon = \operatorname{tg} {}^1/_2\,\gamma \tag{1.4}$$
$$\int_0^1 q(t)\left[\frac{4t}{x+t} K\left(\frac{2\sqrt{xt}}{x+t}\right) + \frac{\varepsilon}{\theta_1} S(x, t)\right] dt = \frac{\varepsilon}{\theta_1}\, w(x), \quad 0 \leqslant x \leqslant 1$$

$$S(x, t) = \frac{4\theta_1 t}{1 + \varepsilon^2 x t} K\left(\frac{2\varepsilon\sqrt{xt}}{1 + \varepsilon^2 x t}\right) + \frac{t}{\sqrt{(1 + \varepsilon^2 x^2)(1 + \varepsilon^2 t^2)}} \times \tag{1.5}$$
$$\left\{\frac{-3 - 2\nu + 4\nu^2}{1 + \nu} + \frac{1}{\pi} \operatorname{Re} \int_0^1 \left(\frac{A}{y^2} + \frac{1}{y^2}\right)\left[U^\circ(y, x, t) + U(y, 2 \operatorname{arc\,tg} \varepsilon x,\right.\right.$$
$$\left.\left.\pi - 2 \operatorname{arc\,tg} \varepsilon t)\right] dy\right\}, \quad q(t) = \frac{4\varepsilon^2 \sigma^\circ(t)}{(1 + \varepsilon^2 t^2)^{3/2} G}$$
$$w(x) = \frac{2}{\sqrt{1 + \varepsilon^2 x^2}}\left[-\frac{1 - \varepsilon^2 x^2}{1 + \varepsilon^2 x^2}\, a + \rho(2 \operatorname{arc\,tg} \varepsilon x)\right]$$

The remaining notation in (1.3) — (1.5) agrees with that used in [1].

The function $S(x, t)$ in (1.5) possesses the same properties, as is easily seen, as the corresponding function (11) in [1].

After regularizing (1.4), based on the solution of the axisymmetric contact problem for an elastic half-space, we obtain a Fredholm integral equation of the second kind to determine the function

$$p(x) = \frac{\pi^2 \theta_1}{\varepsilon}\left[q(x) - \frac{c}{\sqrt{1 - x^2}}\right] \tag{1.6}$$

which agrees in form with (24) in [1]. The solution of the integral equation obtained, constructed by the asymptotic method in [1, 5], can be written in the form of (36) - (39) in [1]. The coefficients A_{2n} are hence now determined only by the shape (1.1) of the stamps

$$2(1 + \varepsilon^2 x^2)^{-1/2} \rho(2 \operatorname{arc\,tg} \varepsilon x) = \sum_{n=0}^{\infty} A_{2n} (\varepsilon x)^{2n} \tag{1.7}$$

The coefficients c_i $(i = 0, 1, 2, 3)$ take the following values:

$$c_0 = \frac{-2-2\nu+3\nu^2}{1+\nu} + \frac{1}{2}\ln 2 + \frac{1}{2}\operatorname{Re} A \int_0^1 \frac{y^{2-\lambda}}{1+y}\,dy - \frac{1}{2}\operatorname{Re} A \int_0^1 \frac{1-y^{2-\lambda}}{1-y}\,dy \qquad (1.8)$$

$$c_1 = (1-16\nu^2)\,\theta_1, \quad c_2 = 4+6\nu+4\nu\,(1-\nu)\,/\,(3-16\nu^2)$$

$$c_3 = -2c_0 + \frac{5}{2} - 4\ln 2 + 6\nu - 32\nu\,(1-\nu)\,(1-2\nu) - 4\operatorname{Re} A \int_0^1 y^{2-\lambda} \times$$

$$\frac{3+3y+y^2}{(1+y)^3}\,dy - 4\operatorname{Re} A \int_0^1 \Big[y^{2-\lambda} - 1 + (2-\lambda)(1-y) -$$

$$\frac{1}{2}(2-\lambda)(1-\lambda)(1-y)^2\Big]\frac{3-3y+y^2}{(1-y)^2}\,dy$$

The remaining notation is the same as in [1]. Numerical values of the coefficients c_i from (1.8) are presented below for different values of the Poisson's ratio ν.

ν	c_0	c_1	c_2	c_3
0	—2.9309	0.1592	4.0000	0.980
0.1	—2.3888	0.1203	5.6224	2.136
0.2	—1.8494	0.0458	6.7104	3.932
0.3	—1.3503	—0·0490	7.1104	5.622
0.4	—0.9216	—0.1490	6.8224	6.677
0.5	—0.5885	—0.2387	6.0000	6.345

The constant c in (1.6) is determined from the relationship (28) in [1]. For stamps without corners this relationship (taking the form (40) from [1] for $c = 0$) serves to determine the quantity $\varepsilon = \operatorname{tg} {}^1/_2\,\gamma$ which characterizes the size of the contact domains.

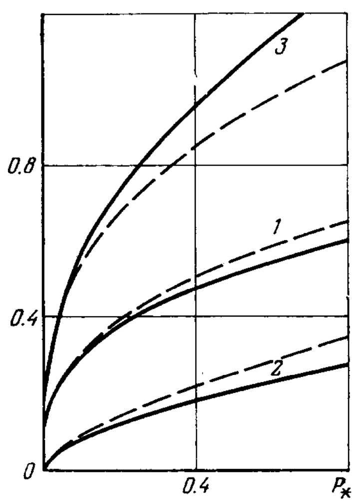

Fig. 2

The magnitude of the force P (Fig. 1) acting on the stamp can be found from the equilibrium condition for the stamp. For $c = 0$ the resultant Z of the contact pressure, which equals the force P in magnitude, is determined by (41) in [1] with (1.7) and (1.8) taken into account.

2. As in illustration, let us consider the problem of impressing two spherical stamps of radius $R\,(1+\Delta)$ whose surface is given by the equation

$$r = (\sqrt{1-2\Delta+\Delta^2\cos^2\theta} - \Delta\,|\cos\theta|)$$

into an elastic sphere. In this case

$$A_0 = 0, \quad A_2 = \frac{4\Delta}{1+\Delta},$$

$$A_4 = -2\Delta\,\frac{3+4\Delta-\Delta^2}{(1+\Delta)^3}$$

Dependences of the quantities 2ε, $(1+\Delta)\,\Delta^{-1}\,a$, $(1+\Delta)\,(G\Delta)^{-1}\,\sigma(0)$ on the quantity $P_* = (1+\Delta)\,(R^2G\Delta)^{-1}\,P$ for $\nu = 0.3$, $A_4/A_2 \approx -{}^3/_2$, $c = 0$ correspond to curves 1, 2 and 3 in Fig. 2. The dashed lines represent the corresponding dependences obtained on the basis of the Hertz solution. A comparison shows that use of the Hertz solution results in substantial errors for large contact domains.

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NEW EULER STABILITY CRITERION FOR A VISCOELASTIC ROD

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A detailed exposition of the mechanical results announced in [1] is given below.

Let us suppose that a thin viscoelastic variable-section rod of finite length l is subjected to weak bending, under the action of longitudinal compressive force P, and under the influence of a slowly varying external transverse load $p(x, t)$.

Then the deflection $y(x, t)$ of the rod axis is described by the following boundary value problem [2 — 4]:

$$-\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 y}{\partial x^2}\right) - P\frac{\partial^2 y}{\partial x^2} - P\int_0^t K(t,\tau)\frac{\partial^2 y}{\partial x^2}d\tau = \tag{1}$$

$$-p(x, t) - \int_0^t K(t,\tau)\,p(x,\tau)\,d\tau, \quad 0 \leqslant x \leqslant l, \quad 0 \leqslant t < \infty$$

$$U_i[y] = 0, \quad i = 1, 2, 3, 4 \tag{2}$$

Here the notation introduced in [1], and the conditions imposed on the moment of inertia $I(x)$, the creep kernel $K(t, \tau)$ and the left sides of the boundary conditions $U[y]$, are retained. We also proceed from the definition of Euler stability and the critical value of the force P contained in [1]. (Another approach to this question is contained in [5]).

The purpose of this paper is to obtain a lower bound and an exact formula for the critical value of the force P under substantially more general conditions than in [4]. Theorem 1 from [1] on the spectrum of the Volterra operator V

$$(Vf)(t) = \int_0^t K(t,\tau)\,f(\tau)\,d\tau, \quad 0 \leqslant t < \infty \tag{3}$$